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Research Report CCS 428

A CHANCE-CONSTRAINED APPROACH TO

BANK DYNAMIC BALANCE SHEET MANAGEMENT

by

A. Charnes
J. Gallegos
S. Yao

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CENTER FOR CYBERNETIC STUDIES

A. Charnes, Director
Business-Economics Building, 203E
The University of Texas at Austin
Austin, Texas 78712
(512) 471-1821

ABSTRACT

The dynamic balance sheet management model formulated deterministically by Chambers and Charnes was extended stochastically by Cohen and Thore to a two-stage LPU^2 model. This paper develops a chance-constrained model which compares well with the Cohen and Thore example but in addition is easily extendable to formulations of realistic size, as the C-T model is not, and with the availability of dual evaluators.

KEY WORDS

Dynamic balance sheet management

Two-stage LPU^2

Chance-constrained Programming

Intertemporal analysis of bank portfolios

A CHANCE-CONSTRAINED APPROACH

TO

BANK DYNAMIC BALANCE SHEET MANAGEMENT

1. Introduction

Traditional asset management is commonly considered by many bankers to mean a kind of static and quasi-independent portfolio management. They focus only on the allocation of funds to various categories to provide adequate liquidity, serve the credit needs of the bank's customers and maximize short run income from investments. The interactions among the various segments of the investment portfolio, the lending policy of the bank and the expectation of making loans in the future are not adequately represented in their view of the portfolio management problem as one of trying to maximize the current yield of the funds allocated to the investment portfolio.

During the past two decades, active management of the entire asset side and liabilities and capital accounts of the bank's balance sheet has become an important part of portfolio management. In order to obtain the maximum benefits for the bank, we must be able to take account of the interdependencies among all the various

balance sheet accounts. This is also why the term "balance sheet management" is more appropriate than either portfolio management or asset management.

Indeed "dynamic balance sheet management" better conveys the idea that optimal decisions regarding the investment securities cannot be made in a static or short-term context.

Proper financial planning and control by a bank involves both long term planning with a time horizon spanning--say ten future years ahead--as well as short term operational management planning. Any predictions of the future might well prove to be wrong. Thus, new predictions should be made continually on the basis of the latest available information. The process of planning and replanning long term and short term is the essence of sound bank financial management [9]!

The most successful technique that has actually been developed and implemented in American commercial banks to help senior executives explore and understand the trade-off relationships in dynamic balance sheet management is that of the large-scale deterministic LP model. From both a theoretical and realistic viewpoint, however, it would be desireable to be able to consider economic events such as loan demand, deposit levels and interest rates as random variables described by some joint

probability distribution. A major problem with such an approach might be the detail and complexity of the informational requirements of the resulting model which might thereby be impossible to implement constructively. It seems wise therefore to look for a constructively possible means of bringing into the analytical picture the major features of the uncertainties within which the dynamic balance sheet management process has to be carried out.

K. J. Cohen and S. Thore presented a paper in 1970 [10] in which they developed a two-stage linear programming under uncertainty (LPU^2) model to comprehend the effects of uncertainties in optimal dynamic balance sheet management. Unfortunately, this two-stage LPU^2 model suffers from several disadvantages. It must prescribe both the actions under all possible arising circumstances and the cost-profit evaluations of all such actions. Thereby the model rapidly becomes so large that computationally very few opportunities and relationships can be considered. By formulating a model in terms of CCP concepts we shall be able to bypass the explicit consideration of every possible state and thereby obtain a more adequate analytical instrument.

Cohen and Thore formulated their two-stage linear programming under uncertainty (LPU^2) model with discrete distribution functions as a stochastic variant of the

basic Chambers and Charnes model [9]. LPU^2 was origin-
ally introduced by Dantzig in 1955 [11], and applied by
Dantzig and Ferguson to a transportation problem in
1956. Cohen and Thore broke up a bank's balance sheet
management process as it reflects upon the liquidity
problem into two stages. During the first stage the bank
fixes the value of the portfolio variables under its
control, such as loans made and investments in various
categories. During the second stage, following stochastic
movements in deposits, the bank responds by making adjust-
ments or by borrowing additional reserves. During the
second stage accumulations of other assets are not made
nor are there positive stochastic accumulations of daily
assets or investments or reductions of loans. Cohen
and Thore posit net penalty costs for not covering reserve
deficits during the second stage.

A major weakness of this model is the assumption
that penalty cost can be known and determined a priori
and that they are independent of the mode of operation of
the system. This weakness can be overcome in a chance-
constrained formulation which does not require such
information or assumptions. Chance-constrained program-
ming (CCP) involves planning for "almost all" situations,
not for every possible situation, since many of these
are not predictable a priori and may be impossible to

treat without invoking new measures not under the bank's control. LPU^2 is a very special instance of CCP.

Our CCP model is also based on the deterministic Chambers and Charnes model. It involves a natural extension from deterministic constraints to chance constraints when random variables are involved and from a deterministic objective to one which is the expected value of a function of random variables. We now proceed to formulate our model.

2. Structure of the Model

(A) The Variables

Following the classifications of the Chambers-Charnes deterministic model, which are reproduced approximately in the Cohen-Thore model, the bank is assumed to have initial holdings of cash, daily assets, short-term and long-term investments, loans, Federal funds purchased, Federal Reserve borrowings, certificates of deposits, demand deposits, savings deposits, and capital and reserves, etc.

Knowing these and taking certain actions such as change the daily assets, purchase or sell short-term and long-term investments, issue certificates of deposits, at the beginning of the period, Cohen and Thore obtain results at the end of the period which are acted or to give the

terminal balance sheet results in accordance with penalty costs or horizon constraints or investments and other financial position considerations.

Instead of the C-T procedure, in our CCP representation we do <u>not</u> employ actions after the emergence of the random variable yields or other similar sample information just end of penalty costs, we require desired posture conditions to hold at least a prescribed level of probability.

For us let

X_1= amount of daily assets to have at the end of the period

X_2= amount of short-term investments to have at the end of the period

X_3= amount of long-term investments to have at the end of the period

X_4= amount of loans to have at the end of the period

X_5= amount of Federal funds to be purchased to have at the end of the period

X_6= amount of borrowing from the Federal Reserve to have at the end of the period

X_7= amount of certificates of deposit to hold at the end of the period

Besides these decision variables, we designate as follows the following random variables,

y_1 = amount of emergent cash holdings at the end of the period

y_2 = amount of emergent demand deposits at the end of the period

y_3 = amount of emergent savings deposits at the end of the period

y_4 = amount of emergent capital and reserves at the end of the period

The X_i are required to be and the y_i are non-negative.

Since our decision variables are not functions of the observed sample values of the random variables, our class of stochastic decision rules is the so-called "zero-order" decision rule class.

(B) The Constraints

We now consider what constraints to impose on the bank's balance sheet position. Since some terminal variables are stochastic, those constraints involving them will be chance constraints.

(1) Cash Reserve Requirements Constraint

The bank must maintain at least the cash balances specified by the Federal Reserve System's legal minimum reserve requirements. These depend on both the size and composition of the bank's deposits, and savings deposits at the end of the period, respectively.

The terminal holdings of cash, y_1, may be obtained taking account of decision variable actions by adding cash

inflows to the initial holdings and subtracting all cash outflows. All interest revenues and costs are assumed to be booked on the cash accounts of the current period. Thus, we have

$$(*) \quad y_1 = -\sum_{i=1}^{4} (1-e_i)X_i + \sum_{i=5}^{7} (1-e_i)X_i + y_2 + (1-d_3)y_3 - g + B_6$$

where e_i denotes yield or cost rate of type i

d_3 denotes the interest rate on saving deposits

g denotes the fixed operating cost

B_6 denotes the initial capital and reserves

The Federal Reserve requirements impose certain proportionate contributions a,b,c of the minimum reserve requirements of certificates of deposits, demand deposits, and savings deposits, respectively toward the cash balance. This balance condition may be rendered, if all quantities are deterministic, as

$$y_1 \geq aX_7 + by_2 + cy_3$$

But now y_1, y_2, y_3 are random variables. Thus we

impose instead the chance constraint:

$$P(Y_1 \geq aX_7 + bY_2 + cY_3) \geq \alpha \qquad \ldots(1)$$

where "P" denotes the probability operator and

$$0 < \alpha \leq 1$$

(2) Lower Limit on Daily Assets Constraint

A bank wants to make optimal use of the facilities of the Federal Funds market. Generally one finds that a good general policy is to operate always on both sides of the market. To do this, the bank must sell a certain part of the amount it purchases in the market.

Let ϵ be the fraction of the amount the bank purchases from the market. This policy condition may be rendered as

$$X_1 \geq \epsilon X_5 \qquad \ldots(2)$$

(3) Upper Limit on Federal Funds Purchased Constraint

On the basis of the bank's experiences and contacts, it may wish to specify an upper limit to the amount of Federal funds that it can plan to purchase, thus

$$X_5 \leq K \qquad \ldots(3)$$

where K is the specified upper limit

(4) Lower Limit on Short-term Investments Constraint

The deposit uncertainty relates to the various possible outcomes for the level of deposits in the period. A minimal liquidity buffer is needed to provide for the random variations in deposits.

Let ρ be the fraction of the demand deposits that must be held as short-term investments. Since y_2 is a random variable, we may express this liquidity buffer condition by the chance constraint:

$$P(X_2 \geq \rho y_2) \geq \beta \qquad \qquad \ldots(4)$$

where $\quad 0 < \beta \leq 1$

(5) Upper Limit on Certificates of Deposit Constraint

Management does not wish to become unduly dependent upon certificates of deposit as a source of funds. To represent this policy, we shall limit certificates of deposit to at most the fraction v of the sum of all deposits. Thus, we have

$$X_7 \leq v(X_7 + y_2 + y_3)$$

if these quantities were deterministic.

Since Y_2, Y_3 are random variables, we employ instead the chance constraint:

$$P[v(y_2+y_3) \geq (1-v)x_7] \geq \lambda \qquad \ldots(5)$$

where $\qquad 0 < \lambda \leq 1$

(6) Lower Limit on Total Investments Constraint

The Federal Reserve Board requires that a certain position u of all liabilities and capital must be covered by investments. Assuming that all investments are U.S. government securities, we may regard this as a government-to-assets constraint, which reflects management's need to conform to this externally imposed standard. Let Y_4 denote the holdings of capital and reserves at the end of the period. It may be obtained by adding net income earned during the period to the initial capital and reserves and deducting the operating cost. Thus,

$$(**) \quad y_4 = \sum_{i=1}^{4} e_i X_i - \sum_{i=5}^{7} e_i X_i - d_3 y_3 - g + B_6$$

Deterministically, we would have the constraint:

$$X_2 + X_3 \geq u(x_5 + x_6 + x_7 + y_2 + y_3 + y_4)$$

where u denotes the fraction of all liabilities and capital must be covered by investments.

Since y_2, y_3, y_4 are random variables, we employ the chance constraint:

$$P[x_2+x_3-ux_5-ux_6-ux_7 \geq u(y_2+y_3+y_4)] \geq \gamma$$

$$0 < \gamma \leq 1$$

or, equivalently

$$P[y_2+y_3+y_4 \leq \frac{x_2+x_3}{u} - (x_5+x_6+x_7)] \geq \gamma \qquad \ldots(6)$$

(7) Loans Demand Constraint

The bank wishes to service the loans demand, z. Since z is a random variable not known in advance, we express the bank's desire to service the loans as a chance constraint which states that the bank's operations are to be such that the bank is able to meet the loans demand at least probability δ. Thus, we have

$$P[X_4 \geq z] \geq \delta \qquad \ldots(7)$$

where $\qquad 0 < \delta \leq 1$

(C) The Objective Function

We shall take as the bank's objective the maximization of net return on loans, short and long term investment, daily assets, less interest paid on

deposits, Federal funds, Federal borrowings as well as operating costs, etc. over the time period.

To express the expected net income function, we need to multiply all items by their yield or cost rates, respectively.

Let e_i denote the yield or cost rate of type i in the period. Here, we assume that the e_i are known in advance.

Let g denote the fixed operating cost in the period.

Let d_3 denote the interest rate on savings deposits.

Then, the objective of maximizing expected net income may be rendered as:

$$\text{Max} \sum_{i=1}^{4} e_i X_i - \sum_{i=5}^{7} e_i X_i - d_3 E(y_3) - g$$

where "E" denotes the mathematical expectation operator.

(D) Summary of the Model

Our chance-constrained model may now be formulated as:

$$\text{Max} \sum_{i=1}^{4} e_i X_i - \sum_{i=5}^{7} e_i X_i - d_3 E(y_3) - g$$

subject to

(1) $P(y_1 \geq ax_7 + by_2 + cy_3) \geq \alpha$

(2) $X_4 \geq \epsilon X_5$

(3) $X_5 \leq K$

(4) $P(X_2 \geq \rho y_2) \geq \beta$

(5) $P(y_2 + y_3 \geq \frac{1-v}{v} x_7) \geq \lambda$

(6) $P[y_2 + y_3 + y_4 \leq \frac{x_2 + x_3}{u} - (x_5 + x_6 + x_7)] \geq \gamma$

(7) $P(X_4 \geq z) \geq \delta$

where $X_i \geq 0$ $\qquad\qquad$ $i = 1, 2, \ldots\ldots 7$

(E) The Deterministic Equivalent Problem

We next proceed to reduce these chance-constraints to corresponding deterministic equivalents for the case of zero-order decision rules.

From constraint (1) and "*", we have

$$P[-\sum_{i=1}^{4}(1-e_i)X_i + \sum_{i=5}^{7}(1-e_i)X_i - ax_7 - g + B_6 \geq -(1-b)y_2$$

$$-(1-d_3 - c)y_3] \geq \alpha$$

$$\Rightarrow P[(1-b)y_2 + (1-d_3-c)y_3 \leq ax_7 + g - B_6 + \sum_{1}^{4}(1-e_i)X_i - \sum_{5}^{7}(1-e_i)X_i]$$

$$\leq 1-\alpha$$

Assuming that y_2, y_3 have means $E(Y_2)$, $E(Y_3)$ and variances $V(Y_2)$, $V(Y_3)$, respectively, constraint (1) becomes

$$P\left[\frac{(1-b)y_2+(1-d_3-c)y_3-E((1-b)y_2)-E((1-d_3-c)y_3)}{\sqrt{V((1-b)y_2+(1-d_3-c)y_3}}\right.$$

$$\left.\leq \frac{ax_7+g-B_6+\sum_1^4(1-e_i)X_i - \sum_5^7(1-e_i)X_i-E((1-b)y_2+(1-d_3-c)y_3)}{\sqrt{V((1-b)y_2+(1-d_3-c)y_3}}\right]$$

$$\leq 1-\alpha$$

Let $Z \triangleq \dfrac{(1-b)y_2+(1-d_3-c)y_3-E((1-b)y_2)-E((1-d_3-c)y_3)}{\sqrt{V((1-b)y_2+(1-d_3-c)y_3}}$

then

$$P[Z_1\leq \frac{ax_7+g-B_6+\sum_1^4(1-e_i)X_i-\sum_5^7(1-e_i)X_i-E((1-b)y_2+(1-d_3-c)y_3)}{\sqrt{V((1-b)y_2+(1-d_3-c)y_3)}}]$$

$$\leq 1-\alpha$$

or

$$Fz_1[\frac{ax_7+g-B_6+\sum_1^4(1-e_i)X_i-\sum_5^7(1-e_i)X_i-(1-b)E(Y_2)-(1-d_3-c)E(Y_3)}{\sqrt{V((1-b)y_2+(1-d_3-c)y_3)}}]$$

$$\leq 1-\alpha,$$

where F is the distribution function for the
random variable Z_1.

Since F is a monotone function, we can invert this
to obtain the deterministic equivalent

$$ax_7+g-B_6+ \sum_1^4 (1-e_i)X_i - \sum_5^7 (1-e_i)X_i - (1-b)E(Y_2)-(1-d_3-c)E(Y_3)$$

$$\leq F_{z_1}^{-1}(1-\alpha) \ \sqrt{V((1-b)Y_2+(1-d_3-c)Y_3)} \qquad \ldots\ldots(1')$$

Next, assuming Y_2 has mean $E(Y_2)$ and variance
$V(Y_2)$ and setting $Z_2 \triangleq \dfrac{Y_2-E(Y_2)}{\sqrt{V(Y_2)}}$, we have

$$P[Z_2 \leq \frac{\frac{1}{\rho}x_2-E(Y_2)}{\sqrt{V(Y_2)}}] \geq \beta$$

Hence,

$$F_{z_2}[\frac{\frac{1}{\rho}x_2-E(Y_2)}{\sqrt{V(Y_2)}}] \geq \beta$$

or,

$$\frac{1}{\rho} x_2 - E(Y_2) \geq F_{z_2}^{-1}(\beta) \ \sqrt{V(Y_2)}$$

and the deterministic equivalent ("d.e."later)
is

$$\frac{1}{\rho} x_2 \geq E(Y_2) + F_{z_2}^{-1}(\beta) \ \sqrt{V(Y_2)} \qquad \ldots(4')$$

Next, from (5), we have

$$P[y_2 + y_3 \leq \frac{1-v}{v} x_7] \leq 1 - \lambda$$

Setting $Z_3 \triangleq \dfrac{y_2 + y_3 - E(y_2 + y_3)}{\sqrt{V(y_2 + y_3)}}$, we have

$$P[\, Z_3 \leq \frac{\frac{1-v}{v} x_7 - E(y_2 + y_3)}{\sqrt{V(y_2 + y_3)}} \,] \leq 1 - \lambda$$

Thus,

$$F_{Z_3} [\frac{\frac{1-v}{v} x_7 - E(y_2 + y_3)}{\sqrt{V(y_2 + y_3)}}] \leq 1 - \lambda$$

and the d. e. is

$$\frac{1-v}{v} X_7 - E(y_2) - E(y_3) \leq F_{Z_3}^{-1}(1 - \lambda) \sqrt{V(y_2 + y_3)} \qquad \ldots \ldots (5')$$

From "**" and (6), we have

$$P[y_2 + y_3 + \sum_1^4 e_i X_i - \sum_5^7 e_i X_i - d_3 y_3 - g + B^6 \leq \frac{x_2 + x_3}{u} - (x_5 + x_6 + x_7)] \geq \gamma$$

or,

$$P[Y_2+(1-d_3)Y_3 \leq -\sum_1^4 e_i X_i + \sum_5^7 e_i X_i + g - B_6 + u^{-1}(x_2+x_3-ux_5-ux_6-ux_7)] \geq \gamma$$

Setting $Z_4 \triangleq \dfrac{Y_2+(1-d_3)Y_3-E(Y_2+(1-d_3)Y_3)}{\sqrt{V(Y_2+(1-d_3)Y_3)}}$, we have

$$P[Z_4 \leq \frac{-\sum_1^4 e_i X_i + \sum_5^7 e_i x_i + g - B_6 + u^{-1}(x_2+x_3-ux_5-ux_6-ux_7) - E(Y_2+(1-d_3)Y_3)}{\sqrt{V(Y_2+(1-d_3)Y_3)}}] \geq \gamma$$

so that

$$F_{Z_4}[\frac{-\sum_1^4 e_i X_i + \sum_5^7 e_i X_i + g - B_6 + u^{-1}(x_2+x_3-ux_5-ux_6-ux_7) - E(Y_2)-(1-d_3)E(Y_3)}{\sqrt{V(Y_2+(1-d_3)Y_3)}}] \geq \gamma$$

and the d. e. is

$$-\sum_1^4 e_i X_i + \sum_5^7 e_i X_i + g - B_6 + u^{-1}(x_2+x_3-ux_5-ux_6-ux_7) - E(Y_2)-(1-d_3)E(Y_3)$$

$$\geq F_{Z_4}^{-1}(\gamma) \ \sqrt{V(Y_2+(1-d_3)Y_3}} \qquad \qquad \dots(6')$$

Finally, from (7), we have

$$P(z \leq x_4) \geq \delta$$

Assuming z has mean $E(z)$ and variance $V(z)$, and

Setting $Z_5 \triangleq \dfrac{z - E(z)}{\sqrt{V(z)}}$, we have

$$P[Z_5 \leq \frac{X_4 - E(z)}{\sqrt{V(z)}}] \geq \delta$$

Thus,

$$F_{Z_5}[\frac{X_4 - E(z)}{\sqrt{V(z)}}] \geq \delta$$

and

$$X_4 - E(z) \geq F_{Z_5}^{-1}(\delta) \sqrt{V(z)}$$

so that the d. e. is

$$X_4 \geq E(z) + F_{Z_5}^{-1}(\delta) \sqrt{V(z)} \qquad \ldots(7')$$

Therefore, the deterministic equivalent model is:

$$\text{Max} \sum_{i=1}^{4} e_i X_i - \sum_{i=5}^{7} e_i X_i - d_3 E(y_3) - g$$

s. t.

(1') $aX_7 + \sum_1^4 (1-e_i)X_i - \sum_5^7 (1-e_i)X_i \leq B_6 - g + (1-b)E(y_2) + (1-d_3-c)E(y_3)$

$\qquad + F_{Z_1}^{-1}(1-\alpha) \sqrt{V((1-b)y_2 + (1-d_3-c)y_3)}$

(2) $-X_1 + \epsilon X_5 \leq 0$

(3) $X_5 \leq k$

(4') $-X_2 \leq -\rho E(y_2) - \rho F_{Z_2}^{-1}(\beta) \sqrt{V(y_2)}$

(5') $X_7 \leq \frac{v}{1-v} E(y_2) + \frac{v}{1-v} E(y_3) + \frac{v}{1-v} F_{Z_3}^{-1}(1-\lambda) \sqrt{V(y_2 + y_3)}$

(6') $\sum_1^4 e_i X_i - \sum_5^7 e_i X_i - u^{-1}(X_2+X_3-uX_5-uX_6-uX_7) \leq g-B_6-E(y_2)-(1-d_3)E(y_3)$

$-F_{z_4}^{-1}(\gamma) \sqrt{V(y_2+(1-d_3)y_3)}$

(7') $-X_4 \leq -E(z) - F_{z_5}^{-1}(\delta)\sqrt{V(z)}$

$$X_i \geq 0 \qquad i = 1,2,\ldots,7$$

Its dual :

Min $[B_6-g+(1-b)E(y_2)+(1-d_3-c)E(y_3)+F_{z_1}^{1}(1-\alpha)$

$\sqrt{V((1-b)y_2+(1-d_3-c)y_3)}] W_1 + KW_3 -[E(y_2)+F_{z_2}^{1}(\beta)\sqrt{V(y_2)}]W_4$

$+[E(y_2)+E(y_3)+F_{z_3}^{1}(1-\lambda) \sqrt{V(y_2+y_3)}]W_5-[B_6-g+E(y_2)+(1-d_3)E(y_3)$

$+F_{z_4}^{-1}(\gamma) \sqrt{V(y_2+(1-d_3)y_3}]W_6 -[E(z)+F_{z_5}^{-1}(\delta)\sqrt{V(z)}]W_7$

$-d_3E(y_3) - g$

    s. t.

$(1-e_1)W_1-W_2$		$+e_1W_6$	$\geq e_1$
$(1-e_2)W_1$	$-W_4$	$+(e_2-u^{-1})W_6$	$\geq e_2$
$(1-e_3)W_1$		$+(e_3-u^{-1})W_6$	$\geq e_3$
$(1-e_4)W_1$		$+e_4W_6-W_7$	$\geq e_4$
$-(1-e_5)W_1+\epsilon W_2+W_3$		$+(1-e_5)W_6$	$\geq -e_5$
$-(1-e_6)W_1$		$+(1-e_6)W_6$	$\geq -e_6$
$-(1-e_7-c)W_1$		$+W_5 +(1-e_7)W_6$	$\geq -e_7$

$$W_i \geq 0 \qquad i=1,\ldots,7$$

Note implicitly that no assumption on the distributions have been made to obtain this deterministic equivalent model.

At this point, solely for comparison with the Cohen-Thore LPU[2] results which employ symmetric discrete distributions, we shall take our random variables to be normally distributed. (In later work, non-symmetric gamma distributions will be employed for some of these). Also, y_2, y_3 and z are to be independent random variables.

3. Computational Example

Here we use the same data as in [47] for a numerical example to compare our and Cohen-Thore results. The known parameters are as shown on Table I.

TABLE I
The Parameters

Variable	Mean	Variance
y_2	305,000	205,000,000
y_3	131,250	12,810,000
z	218,000	108,905,000

$\epsilon = .4$	$v = .2$	$a = .04$
$u = .12$	$\rho = .03$	$b = .165$
$k = 20,000$	$g = 2,500$	$c = .04$

e_1	e_2	e_3	e_4	e_5	e_6	e_7	d_3
.02	.025	.035	.04	.025	.06	.0375	.035

We take $\alpha = \beta = \gamma = \delta = \lambda = .95$

Suppose that the initial balance sheet for the bank is the same as in [47]. We list it in Table II.

For readers' convenience we exhibit the linear programming problem which is the deterministic equivalent of our CCP in Table III. Via the SMULP code and the U.T. dual cyber 170/750 computer system, optimal solutions were obtained within 0.265 seconds.

The optimal decision variables are as shown in Table IVa. We translate them into the optimal balance sheet and profit for the bank as shown in Table V.

The optimal balance sheet items are as follows:

Daily assets = X_1 = $8,000

Short-term investments = X_2 = $9,868

Long-term investments = X_3 = $67,235

Loans = X_4 = $364,417

Federal funds purchased = X_5 = $20,000

Borrowing at Federal Reserve = X_6 = 0

Certificates of deposit = X_7 = $10,299

Demand deposits = $E(y_2)$ = $305,000

Savings deposits = $E(y_3)$ = $131,250

Capital and Reserves = $E(y_4)$ = $59,357   (from "**")

Cash = $E(y_1)$ = $76,386   (from "*")

TABLE II

Initial Balance Sheet for the Bank

Account Name	Notation	Yield or Cost Rate	Holding
Cash	A_1	-	$100,000
Daily assets	A_2	.02	50,000
Short-term investments	A_3	.025	150,000
Long-term investments	A_4	.035	50,000
Loans	A_5	.04	200,000
TOTAL ASSETS			$550,000
Federal funds purchased	B_1	.025	$ 15,000
Borrowing at Federal Reserve	B_2	.06	5,000
Certificates of deposits	B_3	.0375	50,000
Demand deposits	B_4	-	300,000
Savings deposits	B_5	.035	130,000
Capital and Reserves	B_6	-	50,000
TOTAL LIABILITIES AND CAPITAL			$550,000

TABLE III

Variables / Coefficient	X_1	X_2	X_3	X_4	X_5	X_6	X_7	r.h.s.
	.98	.975	.965	.96	−.975	−.94	−.9225	≤ 403,182
	1				−.4			≥ 0
					1			≤ 20,000
		1						≥ 9,868
						1		≤ 102,993
	−.02	8.308	8.298	−.04	−.975	−.94	−.9625	≥ 503,379
				1				≥ 235,183
Objective Function	$.02x_1 + .025x_2 + .035x_3 + .04x_4 - .025x_5 - .06x_6 - .0375x_7 - 7094$							

TABLE IV

a) Optimal Decision Variables

Account Name	Variable	(1)Value ($)	(2)C_{ij}	(1)x(2)
Daily assets	X_1	$ 8,000	.02	160
Short-term investments	X_2	9,868	.025	247
Long-term investments	X_3	67,235	.035	2,353
Loans	X_4	364,417	.04	14,577
Federal funds purchased	X_5	20,000	-.025	(500)
Borrowing at Federal Reserve	X_6	-0-	-.06	-0-
Certificates of deposit	X_7	10,299	-.0375	(386)
Objective Value		9,357		(7094)

b) Optimal Dual Variables

Variable	(3)Value	(4)r.h.s.	(3) x (4)
W_1	.04164	$403,182	$ 16,789
W_2	.0208	-0-	-0-
W_3	-0-	20,000	-0-
W_4	.01042	-9,868	(103)
W_5	.00065	102,993	67
W_6	.0006	-503,379	(302)
W_7	-0-	-235,183	-0-
Objective Value	9,357		(7094)

TABLE V

The Optimal Balance Sheets and Net Income for the Bank

Account Name	Holding
Cash	$ 76,386
Daily assets	8,000
Short-term investments	9,868
Long-term investments	67,235
Loans	364,417
TOTAL ASSETS	$525,906
Federal funds purchased	20,000
Borrowing at Federal Reserve	-0-
Certificates of deposits	10,299
Demand deposits	305,000
Savings deposits	131,250
Capital and Reserves	59,357
TOTAL LIABILITIES & CAPITAL	$525,906
PROFIT	$ 9,357

TABLE VI

Optimal Balance Sheet and Net Income at Different Level

ACCOUNT NAME	(level = 95%)	(level = 90%)	(C-T model / level = 100%)
Cash	$ 76,386	$ 71,866	$ 56,011
Daily Assets	8,000	8,000	8,000
Short-term Investments	9,868	9,709	9,159
Long-term Investments	67,235	66,941	65,866
Loan	364,417	369,718	388,345
TOTAL ASSETS	$525,906	$526,234	$527,381
Federal Funds Purchased	20,000	20,000	20,000
Borrowing at Federal Reserve	-0-	-0-	-0-
Certificate of Deposits	10,299	10,434	10,906
Demand Deposits	305,000	305,000	305,000
Savings deposits	131,250	131,250	131,250
Capital & Reserve	59,357	59,550	60,225
TOTAL LIABILITIES AND CAPITAL	$525,906	$526,234	$527,381
PROFIT	$ 9,357	$ 9,550	$ 10,225

From Table II, IV, V, we see that cash and holding of daily assets, short-term investments are reduced from the initial values during the period. However, the holding of long-term investments and the loan portfolio are considerably increased in volume. For the items of liabilities and capital, we see that the federal funds purchased, demand deposits, savings deposits, capital and reserves all are increased while the holding of certificates of deposit will be reduced and the borrowings from Federal Reserve system must be sold out in order to maximize our profit.

Our results are almost the same as those of Cohen and Thore [10] (aside from what is due to our additional liquidity constraint) and the results come from explicitly setting down chance-constraints including horizon posture ones at specified confidence levels rather than arbitrarily assuming penalty costs which cannot be determined or known a priori. Further, we obtain explicitly (as the dual optimal variables) the opportunity costs associated with each constraint. These are not available from the Cohen-Thore model. Still further, it is easy to investigate the effects of changing the required probability levels.

Here we compare three different levels of probability. We list the optimal balance sheets as shown on Table VI.

From Table VI, we see that the daily assets,
Federal funds purchased, borrowing at Federal Reserves,
demand deposits, and savings deposits remain the same no
matter what the probability levels are. The items of cash,
short-term investments, and long-term investments decrease
as one goes from a probability level of .95 to .90. The
other items including profit move in the opposite direction.

Why then do these trends continue on going back
up to a probability level of 1? Because this level refers
to the results of the Cohen-Thore model which has in
particular no constraints on cash level while our model
requires at least 60% of initial cash in reserve. The
power of this liquidity consideration is also immediately
brought to our attention by the optimal dual variables
("opportunity costs") corresponding to our constraints.
The dual variable for this cash constraint is several
times the magnitude of the other opportunity costs.

From Table VI and the primal solution, we see
plausibly that we should make more loans and absorb more
certificates of deposit in order to obtain more profit
no matter what the probability levels are. This is
consistent with common sense. We also see that the
results seem to be sensitive to loan demand and minimum
cash reserve requirements.

We thus look specifically at the values of the
optimal dual variables shown in Table IVb. The first
constraint on minimum cash reserve requirements has a
dual variable big enough to account for most of the
possible change in objective function value. Thus an
increase in this minimum cash level will significantly
change the objective function upwards. The values of
the dual variables are very small for constraints 4,5,
and 6. Therefore, a small change in the right hand
sides will contribute only a small change in profit.

4. Conclusions

In the preceding section we have illustrated
our model with an example employing the data of the
Cohen-Thore example. We pointed out similarities,
differences and made observations on the sensitivity
analyses available through our formulation . In this
section we discuss several other considerations.

Cohen and Thore allowed only demand deposits and
savings deposits as random variables. In our model we
take loan demand, terminal cash holdings, and terminal
holdings of capital and reserve be stochastic as well
as deposit levels.

Cohen and Thore assumed that movements in demand
and savings deposits were perfectly correlated although

they suggested that it would be more realistic to assume
that the movements of these two types of deposits were
governed merely by a statistically dependent joint
probability distribution. But for their model, if there
are 6 possible discrete outcomes for demand deposits and
savings deposits respectively, then there will be 36
possible joint outcomes. In principle one could specify
any type of statistical dependence one wished. However,
one would face the need to specify the probabilities of
all joint events. This rapidly would become an impossible
burden as the number of discrete outcomes and the number
of factors affecting outcomes increased. In our model,
we can easily employ continuous probability distributions
with dependence specified through a few parameters. It is
also easy thereby to study the effects of changes in
these parameters.

A bank normally plans its liquidity position to
enable it to satisfy increases in loan demand. Such loan
demand is not deterministic but is rather stochastic, i.e.
it should be represented as a chance-variable and involved
in chance-constraints. Our model does represent future
loan demand in stochastic fashion. The Cohen-Thore model
does not treat it so and thereby cannot draw conclusions
which properly reflect its stochastic nature.

Up to now, we have assumed that all interest
rates and yields are deterministic and known in advance.
Also CCP model could be extended to a "control" model such
as in Charnes and Stedry [7] for continuing operations
and applications.

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The dynamic balance sheet management model formulated deterministically by Chambers and Charnes was extended stochastically by Cohen and Thore to a two-stage LPU^2 model. This paper develops a chance-constrained model which compares well with the Cohen and Thore example but in addition is easily extendable to formulations of realistic size, as the C-T model is not, and with the availability of dual evaluators.

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